Exam Symmetry in Physics

Date	April 23, 2010
Room	X 5113.0201
Time	9:00 - 12:00
Lecturer	D. Boer

- Write your name and student number on every separate sheet of paper
- You are not allowed to use the lecture notes, nor other notes or books
- All subquestions (a, b, etc) of the three exercises have equal weight
- Answers may be given in Dutch
- Illegible handwriting will be graded as incorrect
- Good luck!

Exercise 1

Consider the cyclic group C_5: gp$\{c\}$ with $c^5 = e$.

(a) Determine the order of the elements of C_5.

(b) Write down all proper invariant subgroups of C_5.

(c) Show that $C_5 \cong \mathsf{Z}/5\mathsf{Z}$, where Z denotes the set of all integers.

(d) Construct the character table of C_5.

(e) Construct the three-dimensional vector representation D^V for the generator c of C_5.

(f) Decompose D^V into irreps of C_5.

(g) Decompose the regular representation of C_5 into irreps.

Exercise 2

Consider the group $SU(n)$ of unitary $n \times n$ matrices with determinant 1.

(a) Show that the Lie algebra of $SU(n)$ consists of all traceless anti-hermitian $n \times n$ matrices.

(b) Count the dimension of this Lie algebra.

(c) Use Schur's lemma to deduce that the center of $SU(n)$ is isomorphic to $\mathbb{Z}_n$ (it is allowed to assume that the defining rep is an irrep).

(d) Find the center of $SU(2)$ by explicitly deriving the 2×2 matrices that commute with the Pauli matrices.

(e) Consider the subgroup $SO(n)$ of orthogonal matrices in $SU(n)$. Show whether this is an invariant subgroup or not.

(f) Give an example of an application of the group $SU(n)$ in physics for a particular $n \geq 2$.

Exercise 3

Consider the following Lorentz transformation:

$$\begin{aligned} x' &= x \\ y' &= y \\ z' &= z\cosh\chi - t\sinh\chi \\ t' &= -z\sinh\chi + t\cosh\chi \end{aligned} \tag{1}$$

(a) Write down the four-dimensional representation of the generator K_3 of this transformation.

(b) Show that upon exponentiation this four-dimensional rep of K_3 indeed yields the above transformation (1).

(c) Demonstrate whether the set of transformations in (1) for all χ forms a group.

(d) Explain whether the set of all pure Lorentz transformations (boosts) forms a group.

(e) Explain the role of the metric tensor in determining the symmetry group of space-time.